

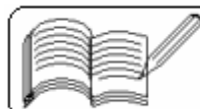
## Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

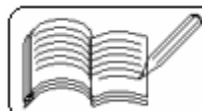
## SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. 15

1. Find the value of a, if P (2, 3) is circumcentre of the triangle with vertices A (a, 6), B (5, 1) and C (4, 6).  
(a) -4      (b) 1      (c) 4      (d) 0
2. Find  $\alpha$  if a line  $x + y + 1 = 0$  is converted in the form of a line  $x \cos\alpha + y \sin\alpha = p$ .  
(a)  $\frac{\pi}{4}$       (b)  $\frac{3\pi}{4}$       (c)  $\frac{5\pi}{4}$       (d)  $\frac{7\pi}{4}$
3. If the circle  $x^2 + y^2 + 4x + Ky - 4 = 0$  touches both the axes, then find K.  
(a)  $\pm 8$       (b)  $\pm 4$       (c)  $\pm 2$       (d)  $\pm 1$
4. Obtain the equation of a parabola having focus (0, -2) and the equation of directrix  $y = 2$ . The vertex of the parabola is (0, 0).  
(a)  $x^2 = -8y$       (b)  $y^2 = 8x$       (c)  $x^2 = 8y$       (d)  $y^2 = -8x$
5. Find the radius of a director - circle of an ellipse  $4x^2 + 9y^2 = 36$ .  
(a)  $\sqrt{5}$       (b)  $\sqrt{13}$       (c)  $\sqrt{10}$       (d) 5
6. If  $|\vec{a}| = 10$ ,  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = 12$ , then find  $|\vec{a} \times \vec{b}|$   
(a) 12      (b) 14      (c) 16      (d) 18
7. Find magnitude of projection of vector  $\vec{i} + \vec{j} + \vec{k}$  on  $\vec{j}$ .  
(a) -1      (b) 0      (c) 1      (d) 2
8. Find the measure of the angle between planes  $\vec{r} \cdot (1, 2, 1) = 1$  and  $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ .  
(a)  $\frac{\pi}{6}$       (b)  $\frac{\pi}{3}$       (c)  $\frac{\pi}{4}$       (d) none of them



9. Find  $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{3}} - 1}{x}$   
(a) 0      (b) 1      (c)  $\frac{1}{3}$       (d) none of these
10. Find  $\frac{d}{dx} \left[ \tan^{-1} \left( \frac{1 - \cos x}{1 + \cos x} \right)^{\frac{1}{2}} \right] : \pi < x < 2\pi$ .  
(a) 0      (b)  $\frac{1}{2}$       (c)  $-\frac{1}{2}$       (d) 1
11. Find c applying Rolle's theorem to  $f(x) = 1 + \sin x, x \in [0, \pi]$ .  
(a) 0      (b)  $\frac{\pi}{4}$       (c)  $\pi$       (d)  $\frac{\pi}{2}$
12. Evaluate :  $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ .  
(a)  $\frac{\pi}{12}$       (b)  $\frac{\pi}{6}$       (c)  $\frac{\pi}{3}$       (d)  $\frac{2\pi}{3}$
13. Find the area of the region bounded by the curve  $y = \tan x$ , X-axis and the lines  $x = 0$  and  $x = \frac{\pi}{4}$ .  
(a)  $\log 2$       (b)  $\frac{3}{2} \log 2$       (c)  $\frac{1}{2} \log 2$       (d)  $2 \log 2$
14. Determine the degree of the differential equation  $\frac{d^2y}{dx^2} + 3 \left( \frac{dy}{dx} \right)^2 = x^2 \log \left( \frac{d^2y}{dx^2} \right)$ .  
(a) 1      (b) 2      (c) 0      (d) not defined
15. A stone falls from a tower of height 40 m. What will be its velocity when it reaches the ground level.  
(a) 14 m/s      (b) 28 m/s      (c) 21 m/s      (d) 7 m/s

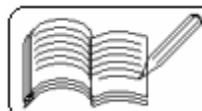


## SECTION B

- Answer the following 16 to 30 questions. Each question carries one mark.

15

16. Find the point A on the X-axis which is at the distance of 5 units from point B (2, -3).
17. Obtain the equation of a circle which touches the X-axis, given that the equations of lines containing two of the diameters of the circle are  $3x - 2y - 5 = 0$  and  $x + y = 5$ .
18. Find the focus of a parabola  $y^2 + 6y - 2x + 5 = 0$ .
19. The equations of the asymptotes of hyperbola are  $3x + 4y = 2$  and  $4x - 3y = 2$ . Find the eccentricity.
20. Find the unit vector in the direction of vector (1, 2, 3).
21. Find the area of a parallelogram, if the diagonals are  $2\bar{i} + \bar{k}$  and  $\bar{i} + \bar{j} + \bar{k}$ .
22. Represent the equation of the line  $\frac{3-x}{1} = \frac{2-y}{3} = \frac{1-z}{4}$  in the vector form.
23. Find the length of a chord cut by sphere  $x^2 + y^2 + z^2 - x - y - z = 0$  on any axis.
24. If  $f'(x) = f(x)$  and  $f(0) = 1$ , then find out the value of  $\lim_{x \rightarrow 0} \frac{f(x) - 1}{x}$ .
25. Evaluate :  $\int x^{4x} (1 + \log x) dx, x > 0$ .
26. Evaluate :  $\int \left( \frac{1+x}{x^2} \right) e^{-x} dx$ .
27. If  $\int_1^k f(x) dx = 47$ ;  $f(x) = \begin{cases} 2x+8, & \text{if } 1 \leq x \leq 2 \\ 6x, & \text{if } 2 \leq x \leq k \end{cases}$  then find k.



28. Find the length of subtangent of  $y = e^{\frac{x}{c}}$ .
29. If a distance of 150 cm. is traveled in 30 seconds with an initial velocity of 10 cm/s, find the constant acceleration (retardation).
30. If the maximum horizontal range is 200 m, find the minimum velocity for that.

<b>SECTION C</b>
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- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

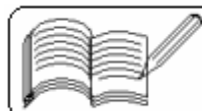
31. A line passing through (2, 4) intersects the X-axis and Y-axis at A and B respectively. Find the equation of the locus of the mid-point of  $\overline{AB}$ .
32. For the parabola  $x^2 = 12y$ , find the area of the triangle, whose vertices are the vertex of the parabola and the two end-points of its latus rectum.
33. Find the equation of the ellipse which is passing through the points (1, 4) and (-6, 1).
34. Find the equation of hyperbola for which the distance from one vertex to two foci are 9 and 1.

**OR**

Find the measure of angle between the asymptotes of hyperbola  $3x^2 - 2y^2 = 1$ .

35. If  $\vec{x} \cdot \vec{y} = \vec{x} \cdot \vec{z}$ ,  $\vec{x} \times \vec{y} = \vec{x} \times \vec{z}$  and  $\vec{x} \neq \vec{0}$ , then prove that  $\vec{y} = \vec{z}$ .
36. If  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ ,  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ , then prove that  $\vec{a} = \pm 2(\vec{b} \times \vec{c})$ , where  $(\vec{b} \wedge \vec{c}) = \frac{\pi}{6}$ .
37. Find the equation of a sphere given that its centre is (1, 1, 0) and that it touches the plane  $2x + 2y + z + 5 = 0$ .

38. If  $y = \tan^{-1} \left( \frac{5x}{1 - 6x^2} \right)$ , then find  $\frac{dy}{dx}$ . OR



$f(x) = [x]$ . Is  $f$  continuous and differentiable at  $x = 1$ ?

39. Find the measure of the angle between the curves  $y = \sin x$  and  $y = \cos x$ ,  $0 < x < \pi$ .

40. Obtain  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ ,  $x \neq \frac{k\pi}{2}$ ,  $\tan x > 0$ . OR Obtain  $\int \frac{dx}{\sin^4 x + \cos^4 x}$ .

**SECTION D**

• Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. A is  $(2\sqrt{2}, 0)$  and B is  $(-2\sqrt{2}, 0)$ . If  $|AP - PB| = 4$ , find the equation of locus of P.

42. Find the equation of the incircle of the triangle formed by the following lines.  
 $x = 2$ ,  $4x + 3y = 5$  and  $4x - 3y + 13 = 0$ .

OR

Get the equation of the circle that passes through the origin and that cuts chords of length 5 on the lines  $y = \pm x$ .

43. Prove by vectors, that if the median on the base of a triangle is also altitude on the base, the triangle is isosceles.

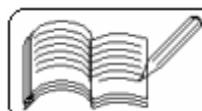
OR

There are two forces  $(2, 5, 6)$  and  $(-1, 2, 1)$  that act on a particle and as a result of which the particle moves from A  $(4, -3, -2)$  and B  $(6, 1, -3)$ . Find the work done.

44. Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect each other and also find the point of intersection.

45. Obtain the equation of a plane that passes through the points  $(2, 3, -4)$  and  $(1, -1, 3)$ , and that is parallel to X-axis.

46. Find  $\lim_{x \rightarrow e^3} \frac{\log x - 3}{x - e^3}$ .



47. Prove that of all the rectangles having the same area, the square has minimum perimeter.

OR

If  $y = ax^3 + bx^2 + cx + 5$  touches X-axis at  $(-2, 0)$  and the slope of the tangent where it meets Y-axis is 3, then find a, b and c.

48. Evaluate :  $\int_0^1 \frac{\log(1+x)}{(1+x)^2} dx$ .

49. Find the area of the region bounded by the curve  $y = 2\sqrt{1-x^2}$  and X-axis. OR

Evaluate :  $\int_2^3 e^{-x} dx$  as the limit of a sum.

50. Solve the differential equation  $x dy + y dx = xy dx$ ,  $y(1) = 1$ .

**SECTION E**

- Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. The equation of the line containing one of the sides of an equilateral triangle is  $x + y = 2$  and one of the vertices of the triangle is  $(2, 3)$ . Find the equations of lines containing the remaining sides of the triangle.

OR

A is  $(1, 3)$  in  $\triangle ABC$  and the lines  $x - 2y + 1 = 0$  and  $y - 1 = 0$  contain two medians of the triangle. Find the co-ordinates of B and C.

52. Find  $\lim_{x \rightarrow 1} \frac{x^n - 1 - n(x-1)}{(x-1)^2}$ ,  $x \neq 1$ .

53. If  $y = \log(1 + \sin x)$ , then prove that  $e^y \cdot \frac{d^2y}{dx^2} + 1 = 0$ .

54. Evaluate :  $\int \left( \frac{2007x + 2008}{2008x + 2007} \right) dx$  OR Evaluate :  $\int \frac{dx}{\sin x + \sec x}$ .

