

- (1) Find the parametric equations of the line passing through A (3, -2) and B (-4, 5) and hence express \overleftrightarrow{AB} , \overrightarrow{AB} and \overline{AB} as sets.

$$\left[\begin{array}{l} \text{Ans: Parametric equations of } \overleftrightarrow{AB} \text{ are } x = -7t + 3, y = 7t - 2, t \in \mathbb{R}. \\ \text{Further } \overleftrightarrow{AB} = \{(-7t + 3, 7t - 2) \mid t \in \mathbb{R}\}, \\ \overrightarrow{AB} = \{(-7t + 3, 7t - 2) \mid t \geq 0, t \in \mathbb{R}\} \\ \text{and } \overline{AB} = \{(-7t + 3, 7t - 2) \mid 0 \leq t \leq 1, t \in \mathbb{R}\} \end{array} \right]$$

- (2) If the length of the perpendicular segment from the origin is 10 and $\alpha = -\frac{5\pi}{6}$, then find the equation of the line.

[Ans: $\sqrt{3}x + y + 20 = 0$]

- (3) If the lines $3x + y + 4 = 0$, $3x + 4y - 15 = 0$ and $24x - 7y - 3 = 0$ contain the sides of a triangle, prove that the triangle is isosceles.

- (4) Find the co-ordinates of the point at a distance of 10 units from the point (4, -3) on the line perpendicular to $3x + 4y = 0$.

[Ans: (10, 5), (-2, -11)]

- (5) A (x_1, y_1) and B (x_2, y_2) are points of the plane. If the line $ax + by + c = 0$ divides \overline{AB} , find the ratio in which it divides \overline{AB} from A.

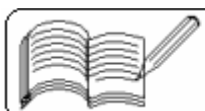
$$\left[\text{Ans: } \lambda : 1 = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}, \quad ax_2 + by_2 + c \neq 0 \right]$$

- (6) If the sum of the intercepts on the axes of a line is constant, find the equation satisfied by the mid-point of the segment of the line intercepted between the axes.

[Ans: $x + y = k$, where $2k = \text{constant sum of the intercepts}$]

- (7) Find k if the lines $kx - y - 2 = 0$, $2x + ky - 5 = 0$ and $4x - y - 3 = 0$ are concurrent.

[Ans: $k = 3$ or -2]



- (8) Among all the lines passing through the point of intersection of the lines $x + y - 7 = 0$ and $4x - 3y = 0$, find the one for which the length of the perpendicular segment on it from the origin is maximum.

[Ans: $3x + 4y - 25 = 0$]

- (9) Prove that the product of the perpendicular distances of the line $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ from the points $\left(\pm \sqrt{a^2 - b^2}, 0 \right)$ is b^2 .

- (10) Prove that if $l m_1 \neq l_1 m$, $n \neq n_1$ and $l^2 + m^2 = l_1^2 + m_1^2$, then the lines $lx + my + n = 0$, $l_1x + m_1y + n_1 = 0$, $lx + my + n_1 = 0$ and $l_1x + m_1y + n = 0$ form a rhombus.

- (11) Prove that the lines $(a^2 - 3b^2)x^2 + 8abxy + (b^2 - 3a^2)y^2 = 0$ and $ax + by + c = 0$, $c \neq 0$ contain the sides of an equilateral triangle whose area is $\frac{c^2}{\sqrt{3}(a^2 + b^2)}$.

- (12) Two lines are represented by $3x^2 - 7xy + 2y^2 - 14x + 13y + 15 = 0$. Find the measure of the angle between them and the point of their intersection.

[Ans: $\frac{\pi}{4}, \left(\frac{7}{5}, -\frac{4}{5} \right)$]

- (13) If the intercepts on the axes by the line $x \cos \alpha + y \sin \alpha = p$ are a and b , prove that $a^{-2} + b^{-2} = p^{-2}$.

- (14) Given $A(2, 2)$, $B(0, 4)$ and $C(3, 3)$, find the equation of (i) the median of triangle ABC through A, (ii) the altitude of triangle ABC through A (iii) the perpendicular bisector of \overline{BC} and (iv) the bisector of $\angle BAC$.

[Ans: (i) $3x + y = 8$, (ii) $3x - y = 4$, (iii) $3x - y = 1$ and (iv) $x = 2$]

- (15) Equations of the two of the sides of a parallelogram are $3x - y - 2 = 0$ and $x - y - 1 = 0$ and one of its vertices is $(2, 3)$. Find the equations of the remaining sides.

[Ans: $x - y + 1 = 0$, $3x - y - 3 = 0$]



(16) A line passes through $(\sqrt{3}, -1)$ and the length of the segment perpendicular to it from the origin is $\sqrt{2}$. Find the equation of the line.

[Ans: $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 4$, $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y = 4$]

(17) Find the equation of a line through $(2, 6)$ if the length of the perpendicular segment to it from the origin is 2.

[Ans: $x = 2$, $4x - 3y + 10 = 0$]

(18) Find the equation of the line which passes through $(3, -2)$ and which makes an angle of 60° with the line $\sqrt{3}x + y = 1$.

[Ans: $y + 2 = 0$, $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$]

(19) Find the equation of the line which passes through $(3, 4)$ and which makes an angle of 45° with the line $3x + 4y = 2$.

[Ans: $x - 7y + 25 = 0$, $7x + y - 25 = 0$]

(20) Find the points on $2x + y = 1$ which are at a distance $\sqrt{5}$ from $(1, -1)$.

[Ans: $(0, 1)$, $(2, -3)$]

(21) Find the points on $2x + y = 1$ which are at a distance 2 from $(1, 1)$.

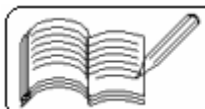
[Ans: $(1, -1)$, $\left(-\frac{3}{5}, \frac{11}{5}\right)$]

(22) A line intersects X- and Y-axes at A and B respectively. If $AB = 10$ and $3OA = 4OB$, then find the equation of the line.

(Ans: $\pm 3x \pm 4y = 24$)

(23) The points $(1, 2)$ and $(3, 8)$ are a pair of opposite vertices of a square. Find the equations of the lines containing its sides and diagonals.

(Ans: $x - 2y + 3 = 0$, $x - 2y + 13 = 0$, $2x + y - 4 = 0$, $2x + y - 14 = 0$,
 $3x - y - 1 = 0$, $x + 3y - 17 = 0$)



(24) An adjacent pair of vertices of a square is $(-1, 3)$ and $(2, -1)$. Find the remaining vertices.

[Ans: $(6, 2)$, $(3, 6)$, $(-2, -4)$, $(-5, 0)$]

(25) Find the equations of the lines passing through $(-2, 3)$ which form an equilateral triangle with the line $\sqrt{3}x - 3y + 16 = 0$

(Ans: $x + 2 = 0$, $x + \sqrt{3}y + 2 - 3\sqrt{3} = 0$)

(26) Area of triangle ABC is 4. The co-ordinates of A and B are $(2, 1)$ and $(4, 3)$. Find the co-ordinates of C if it lies on the line $3x - y - 1 = 0$.

[Ans: $(2, 5)$, $(-2, -7)$]

(27) Find the equation of a line passing through $(2, 3)$ and which contains a segment of length 2 between the lines $2x + y - 5 = 0$ and $2x + y - 3 = 0$.

[Ans: $3x + 4y - 18 = 0$, $x = 2$]

(28) Show that the centroid of triangle ABC lies on the line $21x + 27y - 74 = 0$ if C lies on $7x + 9y - 10 = 0$ and A and B have co-ordinates $(6, 3)$ and $(-2, 1)$.

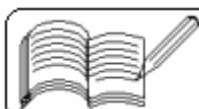
(29) If $\frac{1}{a} + \frac{1}{b} = k$, then prove that all lines $\frac{x}{a} + \frac{y}{b} = 1$ pass through a fixed point.

(30) Prove that if $a + b + c = 0$, and $b^2 \neq ac$, $c^2 \neq ab$ and $a^2 \neq bc$, then the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent and find their point of concurrence.

[Ans: $(1, 1)$]

(31) Find the co-ordinates of the foot of the perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$

[Ans: $\left(\frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2} \right)$]



(32) Without finding the point of intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$, find the equation of the line passing through that point and satisfying the given conditions:

- (i) whose both intercepts are equal,
- (ii) whose sum of the two intercepts is zero, but their product is not zero,
- (iii) whose distance from origin is 13 units and
- (iv) for which the product of both intercepts is -30.

[Ans: (i) $x + y - 17 = 0$, $5x = 12y$, (ii) $x - y - 7 = 0$, (iii) $12x + 5y - 169 = 0$,
(iv) $5x - 6y - 30 = 0$, $5x - 24y + 60 = 0$]

(33) Find the points on the line $3x - 2y - 2 = 0$ which are at a distance 3 units from $3x + 4y - 8 = 0$.

[Ans: $\left(-\frac{1}{3}, -\frac{3}{2}\right)$, $\left(3, \frac{7}{2}\right)$]

(34) Prove that $x^2 - y^2 - 2xy \tan \theta + 2ay \sec \theta - a^2 = 0$ represents a pair of lines and find their point of intersection.

[Ans: $(a \sin \theta, a \cos \theta)$]

(35) Find the measure of the angle between the lines

$$(\cos^2 \alpha - \cos^2 \theta)x^2 - xy \sin 2\theta + (\cos^2 \theta - \sin^2 \alpha)y^2 = 0, \quad 0 < \alpha < \frac{\pi}{4}.$$

[Ans: 2α]

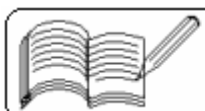
(36) Prove that the difference between the slopes of the lines

$$(\tan^2 \theta + \cos^2 \theta)x^2 - 2xy \tan \theta + y^2 \sin^2 \theta = 0 \text{ is } 2.$$

(37) Prove that the equation of the lines through the origin which makes an angle of measure α with $x + y = 0$ is $x^2 + 2xy \sec 2\alpha + y^2 = 0$ ($0 < \alpha < \frac{\pi}{4}$).

(38) If the equation $x^2 - \lambda xy + 4y^2 + x + 2y - 2 = 0$ represents two lines, then find λ .

[Ans: -4, 5]



- (39) The sides of a triangle are along the lines $x - 2y + 2 = 0$, $3x - y + 6 = 0$ and $x - y = 0$. Find the orthocentre of the triangle.

[Ans: $(-7, 5)$]

- (40) Find the area of the triangle whose sides are along the lines $x = 0$, $y = m_1x + c_1$ and $y = m_2x + c_2$.

$$\left[\text{Ans: } \frac{(c_1 - c_2)^2}{2 |m_1 - m_2|} \right]$$

- (41) Find the area of the parallelogram whose sides are along the lines $y = mx + a$, $y = mx + b$, $y = nx + c$ and $y = nx + d$.

$$\left[\text{Ans: } \left| \frac{(a - b)(c - d)}{m - n} \right| \right]$$

- (42) Find the points on the line $x + 5y - 13 = 0$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$.

$$\left[\text{Ans: } \left(1, \frac{12}{5} \right), \left(-3, \frac{16}{5} \right) \right]$$

- (43) One pair of opposite vertices of a rhombus is $(-2, 5)$ and $(6, 7)$. One of its sides is along the line $x - 2y + 12 = 0$. Find the equations of the lines along which the remaining sides and diagonals of the rhombus lie.

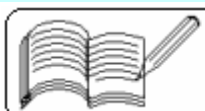
[Ans: $x - 2y + 8 = 0$, $x - 38y + 260 = 0$, $x - 38y + 192 = 0$, $4x + y - 14 = 0$ and $x - 4y + 22 = 0$]

- (44) In triangle ABC, A is $(3, 4)$ and the lines containing two of the altitudes are $4x + y = 0$ and $3x - 4y + 23 = 0$. Find the co-ordinates of B and C.

[Ans: $(-5, 2)$, $(-3, 12)$]

- (45) Find the co-ordinates of the foot of the perpendicular from the point $(a, 0)$ on the line $y = mx + \frac{a}{m}$, $m \neq 0$.

[Ans: $(0, a/m)$]



- (46) Co-ordinates of A in triangle ABC are (1, -2) and the equations of the perpendicular bisectors of \overline{AB} and \overline{AC} are $x - y + 5 = 0$ and $x + 2y = 0$. Find the co-ordinates of B and C.

$$\left[\text{Ans: } (-7, 6), \left(\frac{11}{5}, \frac{2}{5} \right) \right]$$

- (47) In triangle ABC, A is (4, -3) and two of the medians lie along the lines $2x + y + 1 = 0$ and $x + 5y - 1 = 0$. Find the co-ordinates of B and C.

$$[\text{Ans: } (-2, 3), (-4, 1)]$$

- (48) Find the combined equation of the lines through the origin which are perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$.

$$[\text{Ans: } bx^2 - 2hxy + ay^2 = 0]$$

- (49) If the line connecting A ($at_1^2, 2at_1$) and B ($at_2^2, 2at_2$) passes through the point (a, 0), then prove that $t_1t_2 = -1$.

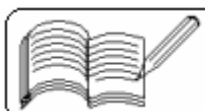
- (50) Prove that the equation of the line passing through A ($a \cos \alpha, b \sin \alpha$) and B ($a \cos \beta, b \sin \beta$) is $\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$.

- (51) Show that the equation of the line passing through ($a \cos^3 \theta, a \sin^3 \theta$) and perpendicular to $x \sec \theta + y \operatorname{cosec} \theta = a$ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.

- (52) Find the equation of the line passing through the origin and cutting off a segment of length $\sqrt{10}$ between the lines $2x - y + 1 = 0$ and $2x - y + 6 = 0$.

$$[\text{Ans: } x - 3y = 0, 3x + y = 0]$$

- (53) If p and p' are the perpendicular distances of the origin from the lines $x \sec \theta + y \operatorname{cosec} \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$ respectively, then prove that $4p^2 + p'^2 = a^2$.



(54) If a perpendicular from origin on a line passing through the point of intersection of $4x - y - 2 = 0$ and $2x + y - 10 = 0$ is of length 2, then find the equation of the line.

[Ans: $x = 2, 4x - 3y + 10 = 0$]

(55) Find the orthocentre of the triangle ABC formed by the three lines $y = a(x - b - c)$, $y = b(x - c - a)$ and $y = c(x - a - b)$.

[Ans: $(-abc, 1)$]

(56) If the line $\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta}$ passing through $A(x_1, y_1)$ meets the line $ax + by + c = 0$ at B, then prove that $AB = \left| \frac{ax_1 + by_1 + c}{a \cos \theta + b \sin \theta} \right|$.

(57) If the line $lx + my + n = 0$ is the perpendicular bisector of \overline{AB} joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then show that $\frac{x_1 - x_2}{l} = \frac{y_1 - y_2}{m} = \frac{2(lx_1 + my_1 + n)}{l^2 + m^2}$.

(58) If in a pair of straight lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, the slope of one line is k times that of the other, then prove that $4kh^2 = ab(1 + k)^2$.

(59) The line $3x + 2y = 24$ meets the Y-axis at A and the X-axis at B. The perpendicular bisector of \overline{AB} meets the line through $(0, -1)$ parallel to X-axis at C. Find the area of the triangle ABC.

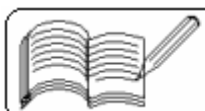
[Ans: 91 sq. units]

(60) A line $4x + y = 1$ through the point $A(2, -7)$ meets the line \overleftrightarrow{BC} whose equation is $3x - 4y + 1 = 0$ at the point B. Find the equation of the line \overleftrightarrow{AC} , so that $AB = AC$.

[Ans: $52x + 89y + 519 = 0$]

(61) Find orthocentre of the triangle whose vertices are $[at_1t_2, a(t_1 + t_2)]$, $[at_2t_3, a(t_2 + t_3)]$ and $[at_3t_1, a(t_3 + t_1)]$.

[Ans: $[-a, a(t_1 + t_2 + t_3 + t_1 t_2 t_3)]$]



(62) Show that if $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then

(i) the equation of the median through A is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \text{and}$$

(ii) the equation of the internal bisector of angle A is given by

$$b \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0, \quad \text{where } b = AC \text{ and } c = AB.$$

(63) Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of the line different from L_2 which passes through P and makes the same angle θ with L_1 .

[Ans: $2(a^2 + b^2)(lx + my + n) - (a^2 - b^2)(ax + by + c) = 0$]

(64) Find the locus of the mid-point of the portion of the variable line $x \cos \alpha + y \sin \alpha = p$, intercepted by the co-ordinate axes, given that p remains constant.

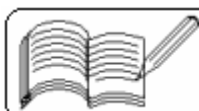
[Ans: $x^{-2} + y^{-2} = 4p^{-2}$]

(65) A variable straight line drawn through the point of intersection of the lines $bx + ay = ab$ and $ax + by = ab$ meets the co-ordinate axes in A and B . Show that the locus of the mid-point of \overline{AB} is the curve $2xy(a + b) = ab(x + y)$.

(66) Show that the quadrilateral formed by the lines $ax \pm by + c = 0$ and $ax \pm by - c = 0$ is a rhombus and that its area is $\frac{2c^2}{|ab|}$.

(67) Suppose $a \neq b$, $b \neq c$, $c \neq a$, $a \neq 1$, $b \neq 1$, $c \neq 1$ and the lines $ax + y + 1 = 0$, $x + by + 1 = 0$ and $x + y + c = 0$ are concurrent, then prove that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$$



(68) The equation of a line bisecting an angle between two lines is $2x + 3y - 1 = 0$. If one of the two lines has equation $x + 2y = 1$, find the equation of the other line.

[Ans: $19x + 22y = 3$]

(69) Prove that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ subtending right angle at the origin are concurrent.

(70) A line cuts X-axis at $A(7, 0)$ and the Y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the X-axis at P and the Y-axis at Q. If AQ and BP intersect in R, find the locus of R. [IIT 1990]

[Ans: $x^2 + y^2 - 7x + 5y = 0$]

(71) Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point $(1, 2)$. [IIT 1990]

[Ans: $7x + y - 9 = 0$, $x - 7y + 13 = 0$]

(72) The sides of a triangle are along the lines $L_i \equiv x \cos \alpha_i + y \sin \alpha_i - p_i = 0$, $i = 1, 2, 3$. Prove that the orthocentre of the triangle is given by

$$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2).$$

(73) A line through $A(-2, -3)$ meets the lines $x + 3y - 9 = 0$ and $x + y + 1 = 0$ at B and C respectively such that $AB \cdot AC = 20$. Find the equation of \overleftrightarrow{AB} .

(74) A line through $A(-5, -4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C, and D respectively. If AC, AB, AD are in H. P., then find the equation of \overleftrightarrow{AB} .

(75) Find the equation of the line passing through the point of intersection of the lines $x + y = 2$ and $3x - y = 2$ and for which perpendicular distance from the origin is the shortest.

