

(1) Show that  $P(a, b+c)$ ,  $Q(b, c+a)$  and  $R(c, a+b)$  are collinear.

(2) Prove that the two lines joining the mid-points of the pairs of opposite sides and the line joining the mid-points of the diagonals of a quadrilateral are concurrent.

(3) If  $(2, 3)$ ,  $(4, 5)$  and  $(a, 2)$  are the vertices of a right triangle, find  $a$ .

[ Ans: 3, 7 ]

(4) Find the circumcentre of the triangle with vertices  $(-1, 1)$ ,  $(0, -4)$  and  $(-1, -5)$  and deduce that the circumcentre of the triangle whose vertices are  $(2, 3)$ ,  $(3, -2)$  and  $(2, -3)$  is the origin.

[ Ans:  $(-3, -2)$  ]

(5) For which value of  $a$  would the area of a triangle with vertices  $(5, a)$ ,  $(2, 5)$  and  $(2, 3)$  be 3 units?

[ Ans: For any  $a \in \mathbb{R}$  ]

(6) Find the area of the triangle whose vertices are  $(l^2, 2l)$ ,  $(m^2, 2m)$  and  $(n^2, 2n)$  if  $l \neq m \neq n$ .

[ Ans:  $|(l - m)(m - n)(n - l)|$  ]

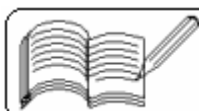
(7) Find the area of the triangle whose vertices are  $(5, 3)$ ,  $(4, 5)$  and  $(3, 1)$  and show that the triangle whose vertices are  $(-2, 2)$ ,  $(-3, 4)$  and  $(-4, 0)$  has the same area.

[ Ans: 3 units ]

(8) Find the area of the triangle with vertices  $(5, 3)$ ,  $(4, 5)$  and  $(3, 1)$  by shifting the origin at  $(5, 3)$ .

[ Ans: 3 units ]

(9) Prove that the mid-point of the segment joining the two points dividing  $\overline{AB}$  from  $A$  in the ratios  $m : n$  and  $n : m$  is the mid-point of  $\overline{AB}$ .



(10) If  $P(1, 2)$  and  $Q(5, 6)$  divide  $\overline{AB}$  from  $A$  in the ratios  $2 : 1$  and  $-2 : 1$ , find the co-ordinates of  $A$  and  $B$ .

[ Ans:  $A(-1, 0)$ ,  $B(2, 3)$  ]

(11) If  $(3, 2)$ ,  $(4, 5)$  and  $(2, 3)$  are three of the four vertices of a parallelogram, find the co-ordinates of the fourth vertex.

[ Ans:  $(5, 4)$ ,  $(3, 6)$ ,  $(1, 0)$  ]

(12) Show that the points  $(2, 3)$ ,  $(4, 5)$  and  $(3, 2)$  can be the vertices of a rectangle and find the co-ordinates of the fourth vertex.

[ Ans:  $(5, 4)$  ]

(13) If the mid-points of the sides of a triangle are  $(4, 3)$ ,  $(5, -1)$  and  $(2, 7)$ , find the vertices of the triangle.

[ Ans:  $(7, -5)$ ,  $(1, 11)$ ,  $(3, 3)$  ]

(14) Find co-ordinates of the centroid, circumcentre and in-centre of the triangle whose vertices are  $(3, 4)$ ,  $(0, 4)$  and  $(3, 0)$ .

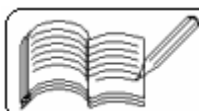
[ Ans:  $\left(2, \frac{8}{3}\right)$ ,  $\left(\frac{3}{2}, 2\right)$ ,  $(2, 3)$  ]

(15)  $A(3, 4)$ ,  $B(0, -5)$  and  $C(3, -1)$  are the vertices of triangle  $ABC$ . Determine the length of the altitude from  $A$  on  $\overleftrightarrow{BC}$ .

[ Ans: 3 units ]

(16) Points  $B(4, 1)$  and  $C(2, 5)$  are given. Find the equation of sets of all points  $P$  in the plane such that  $m\angle BPC = \frac{\pi}{2}$ . Find the set of all such points  $P$ .

[ Ans:  $\{(x, y) \mid x^2 + y^2 - 6x + 6y + 13 = 0\} - \{B, C\}$  ]



(17) If  $A(0, 1)$  and  $B(2, 9)$  are given, find  $C$  on  $\overleftrightarrow{AB}$  such that  $AB = 3 AC$ .

$$\left[ \text{Ans: } \left( -\frac{2}{3}, -\frac{5}{3} \right), \left( \frac{2}{3}, \frac{11}{3} \right) \right]$$

(18) Points  $A(x_1, x_1 \tan \theta_1)$ ,  $B(x_2, x_2 \tan \theta_2)$  and  $C(x_3, x_3 \tan \theta_3)$  are given. If the circumcentre of triangle  $ABC$  is origin and its centroid is  $(x, y)$ , prove that

$$\frac{x}{y} = \frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{\sin \theta_1 + \sin \theta_2 + \sin \theta_3} \quad (0 < \theta_1, \theta_2, \theta_3 < \frac{\pi}{2} \text{ and } x_1, x_2, x_3 > 0)$$

(19) Prove that the mid-points of the sides of any quadrilateral are the vertices of a parallelogram.

(20) If  $D$  and  $G$  are respectively the mid-point of side  $\overline{BC}$  and the centroid in triangle  $ABC$ , then prove that

$$\begin{aligned} \text{(i)} \quad & AB^2 + AC^2 = 2(AD^2 + BD^2) \quad \text{and} \\ \text{(ii)} \quad & AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2) \end{aligned}$$

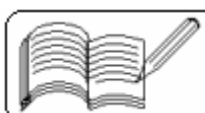
(21) Prove that the coordinates of all three vertices of an equilateral triangle cannot be rational numbers.

(22) If  $A, B, C$  and  $P$  are distinct non-collinear points of the plane, prove that Area of  $\triangle PAB$  + Area of  $\triangle PBC$  + Area of  $\triangle PCA \geq$  Area of  $\triangle ABC$

(23) If  $P$  is a point on the segment joining  $A(3, 5)$  and  $B(-5, 1)$  such that the area of  $\triangle POQ$  is 6 units where  $O$  is the origin and  $Q$  is the point  $(-2, 4)$ , then find the coordinates of  $P$ .

$$\left[ \text{Ans: } (1, 4), \left( -\frac{19}{5}, \frac{8}{5} \right) \right]$$

(24) Prove that the area of the triangle formed by the mid-points of the sides of the triangle is one-fourth that of the original triangle.



- (25) P (-5, 1), Q (3, 5), B (1, 5) and C (7, -2) are four points in the plane. The point A divides the segment  $\overline{PQ}$  in the ratio  $\lambda : 1$  from P. If the area of triangle ABC is 2 units, find the value of  $\lambda$  and also the co-ordinates of A.

$$\left[ \text{Ans: } \lambda=7, A\left(2, \frac{9}{2}\right); \lambda = \frac{31}{9}, A\left(\frac{6}{5}, \frac{41}{10}\right) \right]$$

- (26) If the co-ordinates of A, B and P are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x, y)$  respectively, and if A - P - B, then prove that  $x + y$  lies between  $x_1 + y_1$  and  $x_2 + y_2$ .

- (27) Chord  $\overline{CD}$  is parallel to the diameter  $\overline{AB}$  of a given circle. P is any point on  $\overline{AB}$ . Prove that  $PA^2 + PB^2 = PC^2 + PD^2$ .

- (28) If P is a variable point on the circumcircle of an equilateral triangle ABC, prove that the value of  $AP^2 + BP^2 + CP^2$  is independent of the position of the point P.

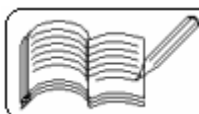
- (29) A straight line  $l$  intersects the lines  $\overleftrightarrow{BC}$ ,  $\overleftrightarrow{CA}$  and  $\overleftrightarrow{AB}$  along the sides of triangle ABC respectively at P, Q and R. Prove that  $\frac{BP}{PC} \cdot \frac{CQ}{QA} \cdot \frac{AR}{RB} = 1$ .

- (30) A variable rod of length  $l$  has one end A on X-axis and another end B on Y-axis. Prove that the equation of the set of points P which divide  $\overline{AB}$  in the ratio 1 : 2 from A is  $9x^2 + 36y^2 = 4l^2$ .

- (31) If A is  $(a \cos \alpha, a \sin \alpha)$  and B is  $(a \cos \beta, a \sin \beta)$ , then find AB.

$$\left[ \text{Ans: } 2 \left| a \sin\left(\frac{\alpha - \beta}{2}\right) \right| \right]$$

- (32) A  $(a, b)$  and B  $(c, d)$  are two points. If  $\overline{AB}$  subtends an angle of measure  $\theta$  at the origin, then prove that  $\cos \theta = \frac{ac + bd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$ .



(33) If  $P(at^2, 2at)$ ,  $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$  and  $S(a, 0)$  are three points, show that  $\frac{1}{SP} + \frac{1}{SQ}$  is independent of  $t$ .

(34) For which value of  $k$  would the points  $(k, 2 - 2k)$ ,  $(-k + 1, 2k)$  and  $(-4 - k, 6 - 2k)$  be distinct and collinear?

[ Ans: - 1 ]

(35) A is  $(-4, 0)$  and B  $(4, 0)$ . Find the locus of a point P such that the difference of its distances from A and B is 4.

[ Ans:  $3x^2 - y^2 = 12$  ]

(36) If the distance between the centroid and incentre of the triangle with vertices  $(-36, 7)$ ,  $(20, 7)$  and  $(0, -8)$  is  $\frac{25}{3}\sqrt{205}k$ , then find the value of  $k$ .

[ Ans:  $k = \frac{1}{25}$  ]

(37) Prove that the locus of in-centre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes is the set

$$S = \{(x, y) \mid x^2 = y^2, x, y \in \mathbb{R}\} - \{(0, 0)\}.$$

(38) Prove that the locus of circumcentre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length  $l$  is the set

$$S = \{(x, y) \mid 4x^2 + 4y^2 = l^2\} - \left\{\left(\frac{l}{2}, 0\right), \left(-\frac{l}{2}, 0\right), \left(0, \frac{l}{2}\right), \left(0, -\frac{l}{2}\right)\right\}.$$

(39) Prove that the locus of centroid of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length  $l$  is the set

$$S = \{(x, y) \mid 9x^2 + 9y^2 = l^2\} - \left\{\left(\frac{l}{3}, 0\right), \left(-\frac{l}{3}, 0\right), \left(0, \frac{l}{3}\right), \left(0, -\frac{l}{3}\right)\right\}.$$

